The Math of Space Travel: Orbits and Conic Sections

JOURNEYS IN FILM
educating for global understanding

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About *Journeys in Film*

Founded in 2003, ***Journeys in Film*** operates on the belief that teaching with film has the power to prepare students to live and work more successfully in the 21st century as informed and globally competent citizens. Its core mission is to advance global understanding among youth through the combination of age-appropriate films from around the world, interdisciplinary classroom materials coordinated with the films, and teachers' professional-development offerings. This comprehensive curriculum model promotes widespread use of film as a window to the world to help students to mitigate existing attitudes of cultural bias, cultivate empathy, develop a richer understanding of global issues, and prepare for effective participation in an increasingly interdependent world. Our standards-based lesson plans support various learning styles, promote literacy, transport students around the globe, and foster learning that meets core academic objectives.

Selected films act as springboards for lesson plans in subjects ranging from math, science, language arts, and social studies to other topics that have become critical for students, including environmental sustainability, poverty and hunger, global health, diversity, and immigration. Prominent educators on our team consult with filmmakers and cultural specialists in the development of curriculum guides, each one dedicated to an in-depth exploration of the culture and issues depicted in a specific film. The guides merge effectively into teachers' existing lesson plans and mandated curricular requirements, providing teachers with an innovative way to fulfill their school districts' standards-based goals.

Why use this program?

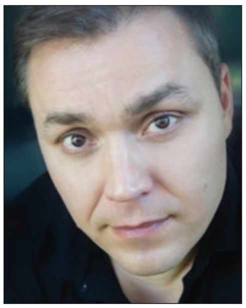
To be prepared to participate in tomorrow's global arena, students need to gain an understanding of the world beyond their own borders. ***Journeys in Film*** offers innovative and engaging tools to explore other cultures and social issues, beyond the often negative images seen in print, television, and film media.

For today's media-centric youth, film is an appropriate and effective teaching tool. ***Journeys in Film*** has carefully selected quality films that tell the stories of young people living in locations that may otherwise never be experienced by your students. Students travel through these characters and their stories: They drink tea with an Iranian family in *Children of Heaven,* play soccer in a Tibetan monastery in *The Cup*, find themselves in the conflict between urban grandson and rural grandmother in South Korea in *The Way Home,* watch the ways modernity challenges Maori traditions in New Zealand in *Whale Rider,* tour an African school with a Nobel Prize-winning teenager in *He Named Me Malala,* or experience the transformative power of music in *The Music of Strangers: Yo-Yo Ma & the Silk Road Ensemble.*

In addition to our ongoing development of teaching guides for culturally sensitive foreign films, ***Journeys in Film*** brings outstanding documentary films to the classroom. ***Journeys in Film*** has identified exceptional narrative and documentary films that teach about a broad range of social issues in real-life settings such as famine-stricken and war-torn Somalia, a maximum-security prison in Alabama, and a World War II concentration camp near Prague. ***Journeys in Film*** guides help teachers integrate these films into their classrooms, examining complex issues, encouraging students to be active rather than passive viewers, and maximizing the power of film to enhance critical thinking skills and to meet the Common Core Standards.

Journeys in Film is a 501(c)(3) nonprofit organization.

A Letter From Theodore Melfi



When you find a career you love, fame is far from your mind. Passion, excitement, and challenging work are instead the driving factors that motivate on a daily basis. Such is the case for Katherine G. Johnson, Dorothy Vaughan, and Mary Jackson—the brilliant trio of African-American women working at NASA in the early 1960s—who helped serve as the brains behind one of the greatest operations in history: the Mercury space missions, culminating in the launch of astronaut John Glenn into orbit.

For decades, until the publication of Margot Lee Shetterly's book *Hidden Figures*, the story of Johnson, Vaughan, and Jackson, NASA's *so-called "human computers,"* went untold. But when their story crossed my path—a story that blurs gender, race, and professional lines—I knew this was a part of history that had to be told. Fifty-five years later, *Hidden Figures* is a rich and moving true story that deserves a spot in our collective consciousness.

The backdrop for the movie is one of the most defining, complex periods in American history: the high-stakes Cold War, the space race, the Jim Crow South and the birth of the civil rights movement. Exploring these historic events serves as a reminder that we must learn from our past experiences while continuing to catapult ourselves forward.

It was also important for me, as a son raised by a single mother and as the father of two daughters, to explore the importance of STEM as a compelling and viable career choice for young girls. The media, cinema, and other public discourse often do society a disservice by not presenting strong, independent women in the fields of science, technology, engineering and math on a regular basis. Drawing attention to these figures, often hidden in plain sight, will hopefully help to chart a new course for female students and change the composition of these vital industries.

At its core, *Hidden Figures* is the story of three remarkable women who overcame every obstacle stacked against them, despite gender, race, and the political landscape of the time. Illuminating this universal experience for the next generation was critical. My goal was to showcase how skill and knowledge are equalizers, how hard work and determination are the cornerstones to every pursuit, and how uniting under a common goal is more powerful than staying divided.

Johnson, Vaughan, and Jackson were pioneers who broke down commonly held perceptions and achieved something phenomenal. Their legacy of persistence serves to empower people of all circumstances and teaches us, as NASA points out in its webpage on Katherine Johnson,

- To love learning.
- To follow your passion.
- To accept the help you're given, and help others when you can.
- To follow new leads and don't give up. Keep trying.
- To go beyond the task at hand; ask questions; be inquisitive. Let yourself be heard.
- To do what you love, and love what you do.

I hope that through the exploration of *Hidden Figures*—and your own passions—you, too, will achieve the seemingly impossible.

Theodore Melfi

Director, *Hidden Figures*

Introducing *Hidden Figures*

Space exploration in the modern age is entering a new phase, replete with private space companies, prospective lunar tourism, and even projected travel to Mars, the closest planet in our solar system. It is fitting, therefore, to pause to look back at the early years of the United States space program, and particularly the early efforts to launch astronauts into orbit, a preliminary step toward a moon landing.

Hidden Figures tells us about a generally unheralded group of women whose brilliance and dedication provided a foundation for the space program—the Black women known as "human computers" who worked at the NASA Center in Langley, Virginia. Faced with obstacles to their own education and to job prospects because of race and gender, these women succeeded in earning places and eventually respect in a workplace dominated by male supervisors and colleagues, many of whom were reluctant to hire women, and marked by segregated facilities, from office to restroom, that reflected the pre-civil rights era.

Katherine Johnson, physicist and mathematician, calculated the orbits, trajectories, and launch windows that would put John Glenn and others into space and bring them back safely. Dorothy Vaughan, another mathematician, became the first African-American supervisor at NASA, learning the computer language FORTRAN on her own and teaching it to her staff. Mary Jackson, an aerospace engineer as well as a mathematician, had to go to court to earn the right to take graduate-level courses at a previously all-white school; she eventually also served as a program officer helping other women succeed at NASA.

Their story is also the story of the world in which they lived and worked—the racism and segregation that made their lives more difficult; the beginnings of the civil rights movement in the South; the Cold War with Russia that gave such impetus to the drive for superiority in space; and the space race itself. The film weaves these events into the dramatic personal stories with skill and accuracy, making it an ideal film for the classroom. It is sure to serve as inspiration to many young women considering a career in science and mathematics.

Hidden Figures has been nominated for many awards, including the Academy Awards, BAFTA, the Golden Globes, the NAACP Image Awards, the Screen Actors Guild, and the African-American Film Critics Association.

Film credits

Director: Theodore Melfi

Screenplay: Allison Schroeder and Theodore Melfi, based on the book with the same title by Margot Lee Shetterly

Producers: Donna Gigliotti, Peter Chernin, Jenno Topping, Pharrell Williams, Theodore Melfi

Actors: Taraji P. Henson, Octavia Spencer, Janelle Monáe, Kirsten Dunst, Jim Parsons, Mahershala Ali, Aldis Hodge, Glen Powell, Kimberly Quinn, Kevin Costner, Olek Krupa

The Math of Space Travel: Orbits and Conic Sections

Enduring Understandings

- In many cases, the orbits of planets and of spacecraft can be described as ellipses or hyperbolas.
- Circles and ellipses are related by scaling. Circles, ellipses, parabolas, and hyperbolas can be generated by slicing certain 3D figures.
- Distances in space are enormous, far beyond everyday human experience. Scientific notation is an effective way to make calculations with the large numbers inherent to orbital calculations.

Essential Questions

- How can the orbits of planets and manmade satellites be represented mathematically? What physical representations can we make of orbital paths?
- How do dilations and scale factors relate geometric shapes to each other and also enable the visualization of large distances?
- How can the large numbers inherent in orbital calculations be handled in an efficient manner?

Notes to the Teacher

Mathematics is at the heart of the film *Hidden Figures*. It is mathematics that supports the ambitions of the three principal characters, mathematics that fills their days in West Computing, and mathematics that brings John Glenn and later astronauts back from their missions. Calculating orbits was particularly important, and so was the ability to handle immensely large numbers in a manageable way. This lesson gives students the opportunity to strengthen their skills in both calculating orbits and managing huge numbers.

Lesson 5 begins with an investigation of calculations that involve exponents, and from there leads students to apply those rules to calculations involving large numbers. The intention is that students will see the need for scientific notation when dealing with astronomical distances and develop an intuitive understanding of the notation through several mental arithmetic exercises.

Exponential arithmetic and scientific notation are together in the same lesson in order to highlight the connections between the two. When students struggle with scientific notation, it is often because they have learned exponential arithmetic by rote, and because they have not been led to see the connections between scientific notation and exponents.

The first part of the lesson is a sequence of problems that lead the students through some (but not all) of the calculation rules for exponential expressions. Scientific notation is nominally a middle school standard in most schools. However, discomfort with exponents, and in particular with scientific notation, is widespread at all ages. Scientific notation is used throughout Lessons 5 and 6. It is recommended that you start with the problems at the beginning of Lesson 5. (Note that the concept of significant digits, though sometimes taught hand-in-hand with scientific notation, is not explicitly used in any of the problems in Lessons 5 or 6.) Additional practice exercises in scientific notation can be found in most high school chemistry or physics textbooks, as well as in many places on the Internet.

Before the class begins, read through the problems on **Handout 1: Problems on Scientific Notation** and **Handout 2: Problems on Conic Sections**. After determining how much class time you have available to cover these topics, select the problems that are most relevant to your course goals, the length of your class period, and the age and ability of your students. There is ample opportunity for differentiation.

Decide on a reasonable number of problems to be worked on for homework in preparation for class discussion. You can expect to have time to discuss no more than about eight such problems in a 50- to 60-minute class period, and usually fewer than that. Some of the shorter problems may require less time, but many of the more challenging problems will require careful discussion. Talk may veer off in any number of directions, as you guide the class to look for connections between ideas. This is, of course, an essential part of the process. Copy the appropriate pages of the handout, marking the problems to be completed for homework. Alternatively, you can copy and paste the assignments for each assignment onto a handout of your own.

The second part of the lesson is designed as a "from first-principles" investigation of ellipses and how they relate to German mathematician and astronomer Johannes Kepler's first and second laws. The goal in many cases is not to construct a mathematical proof of the claims made, but only to examine why they might be reasonable. For a more careful investigation of conic sections, with ideas about how to make them accessible to middle and high school students, an excellent resource is the book *Measurement*, by Paul Lockhart. (You may wish to start with his short book *A Mathematician's Lament*, to understand his background.)

These problems are designed for class discussion, with the students presenting their own work and critiquing that of others. The role of the teacher is to guide the discussion and ask probing questions that may not have occurred to the students.

This lesson avoids the coordinate-centric view of conic sections. A good resource for well-designed problems that examine conic sections and their equations (in Cartesian and polar forms) is the set of math problems used at Phillips Exeter Academy in Exeter, New Hampshire, available here: https://www.exeter.edu/mathproblems.

While it may seem heresy to some, as a STEM educator you are probably already aware that Wikipedia is an excellent, reliable resource for quickly obtaining numerical data about planets and space missions. Some problems require students to obtain orbital data on their own, such as the average distance of planets from the sun. Don't discourage the use of Wikipedia for this. But this is also a great opportunity to have a debate about the difference between "primary source" and "one of your primary resources."

A good tool for enabling discussions in a large class is the portable whiteboard. The American Modeling Teachers Association (AMTA, at http://modelinginstruction.org/) has information on obtaining inexpensive whiteboards, for example the document *Whiteboards*, by Jane Jackson at **http://modeling.asu.edu/modeling/whiteboards2008.doc** and here: **http://legacy.modelinginstruction.org/wp-content/uploads/2013/05/whiteboards2008.doc**. AMTA also has many resources on how to use whiteboards for effective classroom discussion, for example here: **http://modeling.asu.edu/Projects-Resources.html**. Kelly O'Shea also has practical advice on using whiteboards in the classroom here: **https://kellyoshea.blog/tag/whiteboards/**.

Finally, and most importantly: Mathematics works. Science works. It's amazing that some shapes you can draw on a piece of paper are able to describe the motion of absurdly large clumps of atoms (a.k.a. planets) through space with unerring accuracy. Kepler was able to figure all this out just by thinking and calculating, without any computer other than his own brain. He used data mostly collected by Danish astronomer Tycho Brahe, who obtained it just by looking and recording patiently, without a telescope. If you can convey how amazing and exciting that is to your students, then your lesson will have been a success.

A few notes on specific problems on **Handout 2: Problems on Conic Sections**:

- Students see experimentally why it is reasonable to suppose that the circumference of a circle is proportional to its radius in problems 4 to 6. They then come up with arguments (problems 9 and 10) to support a suggested relationship between the circumference and the area of a circle.
- Problems 9, 10, and 11 are designed to strongly suggest to students that decomposing a circle into triangles can be used to justify a method for calculating the area of a circle. Problems 10 and 11 are intentionally designed to highlight a potential case of circular reasoning, if students attempt to justify each area formula by invoking the other.
- The concept of a scale factor, introduced in problem 13, is used throughout Lesson 5 (and Lesson 6) for understanding and making calculations that involve large distances.
- Students learn about and investigate circles and ellipses as slices of a right cylinder. They also examine circles, ellipses, parabolas, and hyperbolas as sections of a cone. The activities with a laser pointer and flashlight (problems 27 and 45), while short, are intended to provide an important kinesthetic route to an understanding of slicing.
- Students are required to know the Pythagorean Theorem in order to relate the semi-major and semi-minor axes of an ellipse to the distance of its focal points from the center of the ellipse.
- Several problems ask students to obtain data about planetary orbits and make calculations with those numbers.

Common Core Standards addressed by this lesson

CCSS.Math.Content.6.G.A.1
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

CCSS.Math.Content.7.G.A.1
Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

CCSS.Math.Content.7.G.A.3
Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

CCSS.Math.Content.7.G.B.4
Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

CCSS.Math.Content.8.EE.A.1
Know and apply the properties of integer exponents to generate equivalent numerical expressions.

CCSS.Math.Content.8.EE.A.3
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

CCSS.Math.Content.8.EE.A.4
Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology

CCSS.Math.Content.8.G.A.3
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

CCSS.Math.Content.8.G.B.7
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Next Generation Science Standards addressed by this lesson

MS-ESS1-3. Analyze and interpret data to determine scale properties of objects in the solar system.

Duration of the Lesson

In a typical 50- to 60-minute class, you can work through an average of six to eight problems per period. Thus, if your students worked through and talked about all problems in this lesson, it would take perhaps nine class periods. That is far too long for most teachers. However, your students do not need to discuss every problem they do. In addition, you do not need to assign every problem. (Be aware, however, that many problems build off previous problems.) Since you are the best judge of your own students, feel free to use all the problems or one of the problems, or anything in between. Or simply use them for reference when writing your own problems and lesson plans.

If you plan to use all the problems and your class period is approximately one hour, you might break them out like this, with one section per day:

Scientific notation

1 to 7

8 to 14

Conics

1 to 8

9 to 16

17 to 20

21 to 28

29 to 34

35 to 43

44 to 47

Assessment

Students' presentations of their approaches to solving the problems

(Having students present their work takes time. However, one significant advantage of their doing so is that formative assessment occurs more naturally. You get continuous information about where your students are in their understanding when they must regularly present their work and answer questions about it.)

Some of the longer problems can also work as extended, written assignments, rather than as topics for classroom debate.

Materials

Handout 1: Problems on Scientific Notation

Handout 2: Problems on Conic Sections.

Procedure

1. Before the class begins, make sure that you have planned the unit thoroughly. See the information in Notes to the Teacher.

2. Ask students to brainstorm all the ways that math was important to the lives of the characters in the film *Hidden Figures*; have them consider not only the women who worked as "human computers" but also the astronauts and others involved in the space program.

3. Tell students that to handle the enormous distances of space, scientists use the tool of scientific notation. Ask students to summarize what they already know about this topic and fill in any gaps they may have. Tell them that this lesson will give them practice in using scientific notation.

4. Distribute **Handout 1** and review the directions with students. Have them check off the problems that you would like them to try to solve for homework. Give them time to begin in class while you circulate to provide guidance. You may wish to allow students to work in pairs in class.

5. In class each day, discuss each problem on scientific notation in turn, having students present their work and asking them questions about their reasoning. Give students time to ask questions until you are satisfied that they understand the basic concepts underlying the problems.

6. When students are secure in their understanding of scientific notation, ask them what the average person means when he or she hears the word "orbit." (A curved path around a celestial body, such as a satellite around the Earth.)

7. Distribute **Handout 2** and begin working the problems together in class.

Handout 1 ▸ P.1

Problems on Scientific Notation

Directions:

Work through the problems below that have been assigned by your teacher. Be sure to show your work. Be prepared to present your solution and explain in class how you arrived at it. Also, bring to class any questions that may have arisen as you worked on these problems.

1. You can simplify a sum of *terms* that share a factor, like $3 \cdot 8 + 7 \cdot 8 + 4 \cdot 8 + 16 \cdot 8$, by counting how many eights there are. In this case, there are 30 eights, so it is equal to $30 \cdot 8$. (Which equals 240, of course.) This process is called *combining like terms*. Use this process to reduce the following to shorter expressions.

1a. $5 \cdot 3 + 6 \cdot 3 + 7 \cdot 3 + 3 \cdot 8$

1b. $18b + 12b - 13b$

1c. $70c + 100d - 20c + 80d$

2. It is possible to justify the process of combining like terms using the *distributive property*. Show how to do so.

3. You can use *exponents* to represent repeated multiplication. For example, $4 \cdot 4 \cdot 4 = 4^3$, $2.3 \cdot 2.3 \cdot 2.3 \cdot 2.3 \cdot 2.3 \cdot 2.3 = (2.3)^6$ and $\frac{5}{7} \cdot \frac{5}{7} \cdot \frac{5}{7} \cdot \frac{5}{7} = \left(\frac{5}{7}\right)^4$. How could you change $7^{40} \cdot 7^{260}$ to a simpler exponential expression? (Note that a calculator won't help much.) What about $7^a \cdot 7^b$?

4. In certain cases, exponents can simplify the process of division. For example, $\frac{4^8}{4^5} = 4^3$. Justify this calculation, using the idea that exponents represent repeated multiplication. Make up two more examples showing this process. Then make up an example of a division problem with exponential expressions that *cannot* be simplified in this way.

5. Use the rule for division from the previous problem to simplify $\frac{4^5}{4^8}$. Can you see another way to simplify this, given that $\frac{4^5}{4^8} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}$? Make up two more examples showing this rule. Using the same method, figure out the best way to define 4^0.

6. How can an expression such as $17 \cdot 8^{354} + 23 \cdot 8^{354}$ be combined into a single term? Make up a similar example, but starting with three terms instead of two.

Handout 1 ▸ P. 2 Problems on Scientific Notation

7. Numbers can be written with a leading or trailing zero to emphasize the location of the decimal point. For example, 743 is 743.0, and .362 is 0.362. Calculate 10^6 and 10^{-4} using a calculator, and write the result with a leading or trailing zero. Explain how you could have done this *without* a calculator by measuring the position of the decimal point relative to the digit 1.

8. "Scientific notation" is used to represent numbers that are very large or very small. These sorts of numbers occur frequently in many fields of science. Examples of scientific notation include 5.3×10^{-6}, -2.7×10^{4}, and 6.02×10^{23}. (Note that × is used to represent multiplication.) On the other hand, 53×10^{-7}, -30^{3}, and 602×10^{21} are *not* scientific notation, even though they are equal in value to each of the former expressions. The reason is one of *convention*: we want to choose a convention that yields one unique way to write every number. Scientific notation makes a somewhat arbitrary choice and chooses a convention that the number before the multiplication sign is between 1 and 10. Look up and write the following values in scientific notation.

8a. the world's current human population

8b. the diameter, in meters, of a single pollen grain of a flower (use a species of your choosing)

8c. the fraction of the federal budget of the United States allocated to NASA in 2013

8d. the size, in meters, of the observable universe

9. "E notation" is a shorthand way of writing "scientific notation," and is common on calculators and computers. In E notation, 5.79×10^{12} is written 5.79E12. On a calculator, the key for entering "E" may be labeled EE, E, EXP, or something similar, depending on the type of calculator you have. (Do not confuse it with the key for e, which is a constant approximately equal to 2.718.) In addition, many calculators have a mode in which all numbers are displayed in scientific notation. Calculate the following without a calculator, writing your results in E notation. Then check your results with a calculator.

9a. 2.0E5 * 3.0E12

9b. 4.0E5 * 3.0E12

9c. 4.0E-7 + 3.0E-7

9d. 2.4E8 / 8.0E2 (Hint: Think of 24/8.)

For the next set of problems all the results should be put in scientific notation.

10. Calculate your current age in seconds.

Handout 1 ▸ P.3

Problems on Scientific Notation

11. The diameter of a carbon atom is approximately 140 picometers. (1 picometer, or 1 pm, is 1×10^{-12} m.) Calculate your own height as measured in carbon atom diameters.

Large distances occur frequently in astronomy and space travel. You would not want to measure your height in carbon atoms, but would use meters. In the same way, we don't want to measure astronomical distance in meters, but instead we use a more appropriate unit. There are several units of distance in common use for astronomical lengths:

- One astronomical unit (1 AU) is the average distance of the Earth from the sun. 1 AU is about $1.496\text{x}10^{11}$ m.
- One light-year (1 ly) is a unit of distance, not a unit of time. It is the distance light travels in one year (in a vacuum).
- One parsec (1 pc) is approximately 3.26 ly. A parsec is defined in terms of the apparent angular motion of stars from the point of view of the Earth as it orbits the sun.

12. What are the distances from the sun of each of the two Voyager probes, measured in AU? What is the distance from the sun to Proxima Centauri, its nearest stellar neighbor, measured in AU?

13. Neptune is approximately 4.5×10^{12} meters from the sun, on average. Look up a typical speed for a passenger jet. If you were to travel at this speed from the sun to Neptune, how many years would your journey take?

14. How many Earths would fit in a cubic light year?

Handout 2 ▸ P.1

Problems on Conic Sections

Directions:

Work through the problems below that have been assigned by your teacher. Be sure to show your work. Be prepared to present your solution and explain how you arrived at it in class. Also, bring to class any questions that may have arisen as you worked on these problems.

1. One way to define a circle is that it is a set of points (or the *locus* of points if you want to be fancy) that are all the same distance from a particular point. This point is called the center of the circle. Figure out a way to draw an accurate circle with a paperclip and two pens or pencils. (A mechanical pencil might not work well for this, by the way.)

2. Figure out a way to draw a circle with a piece of cardboard, a thumbtack, a loop of string, and a pen or pencil.

3. The distance from the center of a circle to its edge is called the radius of the circle. Also, if you draw a line from the center of a circle to its edge, the literal line itself is called a radius. (Note the switch from "the radius" to "a radius" since there are lots of lines you can draw from the center to the edge.) This can be confusing, but unfortunately it happens all the time in geometry. Can you think of any other examples?

4. Take a long piece of light-colored string and any circular object. Your object should be at least a few centimeters across. A diameter of a circle is any line segment stretched all the way across the circle that passes through the center of the circle. Use the string to carefully measure out and cut a diameter of your circle. Fold the diameter in half. With a dry erase or permanent marker, use the folded diameter to mark out eight or so radiuses (not diameters) along your remaining string. (Some people say "radii" instead of "radiuses." Either one is fine.) Measure the number of radiuses that fit around your circle by wrapping the string around it and counting. Compare your number of radiuses with the results of your classmates.

If you were very careful in the previous problem, you probably got something like 6 radiuses, plus a little bit. If you were to repeat this activity, but had a very smooth circle, a very accurate ruler, a string that doesn't stretch, and a very large supply of patience and luck, you might get a value somewhere close to 6.28 radiuses. If you tried this with a bigger circle or a smaller circle, it wouldn't make a difference. You'd always get the same number: about 6.28. For historical reasons, people usually talk about half of this number. Do you recognize it?

Handout 2 ▸ P.2 Problems on Conic Sections

5. Half the number of radiuses around a circle is called by the Greek letter p, which has the symbol π. It is pronounced "pie" by English speakers, as in pecan pie or apple pie, and is spelled "pi." Using methods that definitely do not involve string or scissors, many people have calculated π to much higher accuracy than 3.14. Look up some of the methods people have used over the years to calculate approximations of π. (Just for fun, this site has a million digits of π: http://www.piday.org/million/. The world record for memorizing digits of π is over 70,000 digits which took over 17 hours to recite.)

6. The perimeter of a circle is given by the formula $P = 2\pi r$. Based on your investigation with circles and string, explain why this formula works. (Some people prefer to call the perimeter of a circle its "circumference." You can use it if you like. In that case, the same formula is written $C = 2\pi r$.)

7. The Earth orbits the sun in approximately a circular pattern, and the average distance of the Earth from the sun is about 1.5×10^{11} m. How far does the Earth travel in a year?

8. Out of all the planets that orbit the sun, Venus has an orbit that is closest to a perfect circle. On average, Venus is 0.723 AU from the sun. Venus takes about 225 Earth days to orbit the sun. What is the average speed of Venus in its orbit, in AU per day? In meters per second?

9. Draw a diagram that shows how the inside of a circle can be approximated by a large number of triangles, each with one corner at the center of the circle. If you increase the number of triangles, what happens to the area of each triangle? Does the approximation get better, worse, or stay the same? Why?

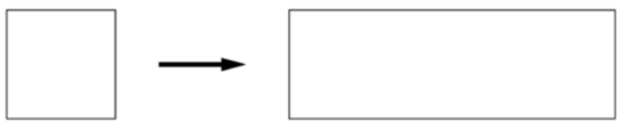
10. One way to calculate the area of a circle is to use the formula $A = \frac{1}{2}C \cdot r$. Explain why this formula works.

11. Another way to calculate the area of a circle is to use the formula $A = \pi r^2$. Explain why this formula works.

12. What is the area enclosed by the orbit of Venus, in square AU? In square meters? (This may seem like a silly thing to calculate, but, as you'll find out soon, Johannes Kepler figured out that it's not actually all that silly.)

Handout 2 ▸ P.3 Problems on Conic Sections

13. You can think of a rectangle as a square that's been stretched out in one direction, without doing anything in the perpendicular direction.

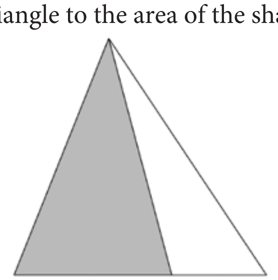


This type of stretching is called a dilation. Using a ruler, measure the dimensions of the square and the rectangle in the figure above. How much was the square stretched horizontally to make the rectangle? Does it make more sense to describe this stretching amount as something you *add* to the square's base, or as something you *multiply* the base by? By the way, the stretching amount is called the "scale factor."

14. Can you consider a squeeze (instead of a stretch) to be a dilation? What sort of number would the scale factor be for a squeeze?

15. What effect does doubling the base of a triangle have on its area? What about doubling its height?

16. In the graphic below, what is the ratio of the area of the unshaded triangle to the area of the shaded triangle?



17. *Scale model of the sun and planets.* The diameter of the sun is 1.39×10^9 meters. The diameter of a basketball typically used in youth leagues is about 0.23 meters. If you were to shrink the sun to the size of a basketball, what would be the scale factor? Find objects that accurately model the size of objects in the solar system, using a basketball (or whatever similar-size ball you have on hand) as the sun. Examples of objects you might use include tennis balls, fruit, marbles, beads, small pieces of candy, grains of salt or sand, or even pepper flakes.

18. *Scale model of the orbits of the planets.* The planet Neptune orbits the sun at an average distance of 30.1 AU. Go to the largest field or flat, open space near your school. (With permission, of course!) A *very* long, quiet hallway will also do, in a pinch. Measure the length of this open space, using a method of your choice. If you model the distance of Neptune from the sun by this length, what is the scale factor? Look up the average distance of each planet from the sun, which is listed under "semi-major axis" in some places. Using the same scale factor you found for Neptune, calculate distances for each planet in your model. Then have some of your fellow students stand at the relative locations of the sun and each of the planets. You might want to bring along some binoculars or a small telescope if you have one! By the way, do the planets ever line up like this, in a single column?

Handout 2 ▸ P.4 Problems on Conic Sections

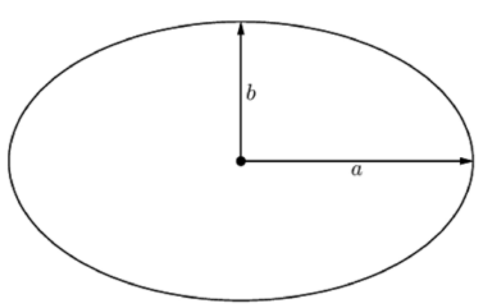
19. Notice that you were not asked to actually use a basketball to model the sun in the previous activity. How big would your scale model of the solar system have to be if you did use a basketball to model the sun?

20. How would your model of the planetary orbits change if you included Pluto?

21. Two dilations. What happens to the area of a triangle if you double the base and double the height? What if you double the base but reduce the height to half its original value?

22. What effect does doubling the radius have on the area of a circle?

23. You can visualize an ellipse as a circle that has been dilated in one direction. Because of this, an ellipse has two radiuses instead of one: a large radius and a small radius.

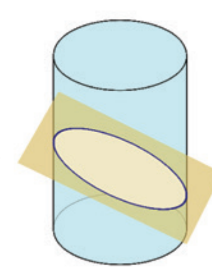


In the diagram here, the large radius is a and small radius is b. (These are also sometimes called the "semi-major axis" and "semi-minor axis," respectively. But those terms are a mouthful, so using large and small radius is easier.) Using a ruler, find the values of a and b, in centimeters. If the ellipse was formed from a circle by dilating it horizontally, what was the scale factor? What if the dilation had been vertical instead?

24. Based on the fact that the area of a circle is $A = \pi \cdot r \cdot r$, you might expect the area of an ellipse to be something like $A = \pi \cdot a \cdot b$. This, in fact, actually is the correct formula. Can you use a dilation to explain why this formula works?

25. What is the area of the ellipse in square centimeters? What is its area in square meters?

You might expect the perimeter of an ellipse to also have a simple formula, just like the area. Unfortunately, it does not. In fact, mathematicians have had to invent an entire subfield of mathematics to study this problem, called "elliptic integration," which has found applications in all sorts of areas unrelated to ellipses.



If you slice a cylinder parallel to its base, the cut part has the shape of a circle. However, if you slice a cylinder diagonally, as in this diagram, the cut part has the shape of an ellipse. Sometimes a cut of this sort is called a "section." (Words such as "bisect," "dissect," and "sector" come from the same Latin root word.) So an ellipse is an example of a "cylindrical section."

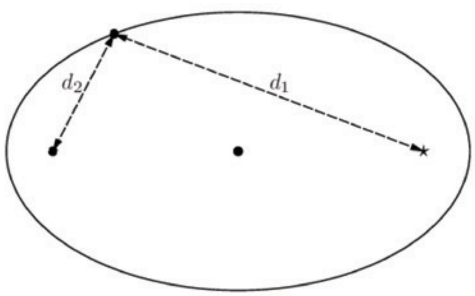
Handout 2 ▸ P.5

Problems on Conic Sections

26. What sort of section do you get if you cut a cylinder perpendicular to its base?

27. Figure out how to make an ellipse by shining a laser pointer on a wall.

Every ellipse has two special points inside of it, called "focal points." Something interesting happens if you take any random point on the outer edge of the ellipse, and measure its distance from each focal point.



If you add these two distances together, you always get the same number. It doesn't matter which point on the edge you pick, as long as it is on the outer edge. (You'll see in a bit why one of the focal points above is a star.)

28. For the ellipse shown above, measure the two distances d_1 and d_2 to the focal points pointed to by arrows on the dashed lines, and add them together. Then pick another arbitrary point on the perimeter of the ellipse, and repeat the same procedure. Do you always get the same sum?

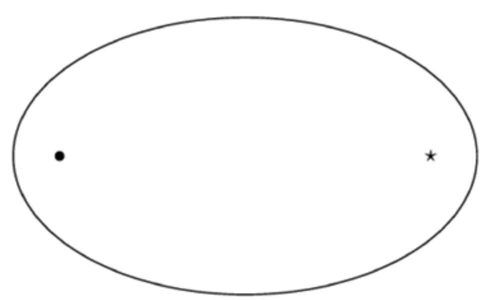
29. Figure out a way to draw an ellipse with a piece of cardboard, two thumbtacks, a loop of string, and a pen or pencil.

30. Pick a random point on the ellipse above problem #28. Using a ruler, verify that $d_1 + d_2 = 2a$. (Consult question #23 for a reminder of what "a" is.) Is this true in general? Support your choice with a diagram.

31. What happens to the focal points of an ellipse as you squeeze an ellipse back to a circle? What happens to the big radius and the small radius?

Johannes Kepler published work in the early 17th century that summarized three observational facts about the motion of the planets. These three facts have since been given the name "Kepler's laws of planetary motion."

The first of these laws is that the orbit of a planet traces out an ellipse, where the sun is located at one of the focal points of the ellipse. (The other focal point doesn't appear to play any role in an orbit. It's almost always just an ordinary, unassuming location in empty space.)

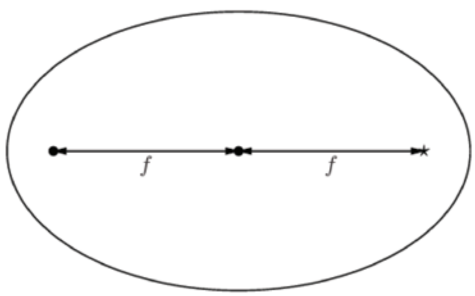


Handout 2 ▸ P.6

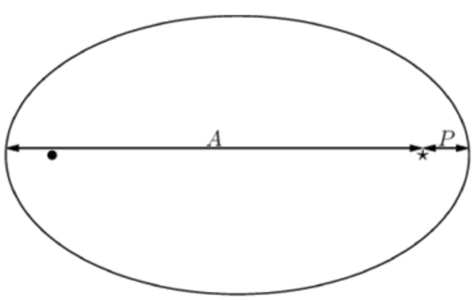
Problems on Conic Sections

32. Perhaps you've heard of black holes. If not, go look them up right now—they're fascinating objects. The region around many black holes is often actually quite bright, because matter falling into it is heated to extremely high temperatures. However, if there is no nearby matter, a black hole is truly black, and is sometimes referred to as "dormant," or "quiescent." If you were able to accurately observe the path of a planet orbiting a dormant black hole in a highly elliptical orbit, would you be able to tell at which focus the black hole was located?

The distance between each focal point and the center of an ellipse doesn't have a commonly used name. Nevertheless, it's a useful quantity. In the ellipse here, this distance is f.



The closest distance a planet gets to the sun is called the "perihelion," and the farthest distance is called the "aphelion." (The ancient Greek word for the sun was *helios*.) In the ellipse below (identical to the one above), the perihelion is labeled P, and the aphelion is labeled A. (Don't confuse it with "a," the big radius of the ellipse.)



33. Using a ruler, verify that $A + P = 2a$ for this ellipse. Is this true in general? Support your choice with a diagram.

34. Using a ruler, verify that $b^2 + f^2 = a^2$. (Consult question #23 for a reminder of what "b" is.) Is this true in general? Support your choice with a diagram. (Hint: Choose a point on the edge of the ellipse that makes $d_1 = d_2$. What else are d_1 and d_2 equal to at this location?)

35. Another way to describe how stretched an ellipse is from a circle is called the "eccentricity," sometimes denoted by the letter e. The eccentricity is defined to be $e = f/a$. What is the eccentricity of the ellipse shown above?

36. What is the eccentricity of a circle? What is the maximum eccentricity of an ellipse?

37. Earth's orbit around the sun is nearly circular. Its perihelion is $P = 0.983$ AU and its aphelion is $A = 1.016$ AU. Calculate the eccentricity of the Earth's orbit.

38. The orbit of the planet Mercury has a moderate eccentricity. Its perihelion is $P = 0.307$ AU, and its aphelion is $A = 0.467$ AU. Calculate the eccentricity of Mercury's orbit.

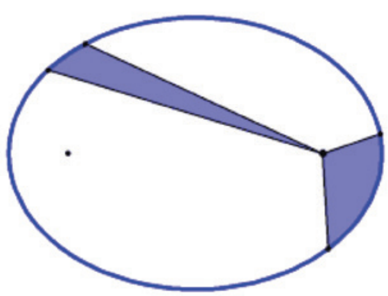
Handout 2 ▸ P.7 Problems on Conic Sections

39. Halley's Comet takes a little more than 75 years to orbit the sun. Its eccentricity is $e = 0.967$. What does this imply about the shape of its orbit?

40. A law in science has nothing to do with the legal system. Look up the difference between a "scientific law" and a "scientific theory." Is the term "law" appropriate for Kepler's first law?

41. Read up on Kepler's life. Where did he get most of his data? What planets or other objects did he base his analysis on? How long did it take him to formulate his laws, based on the data he had?

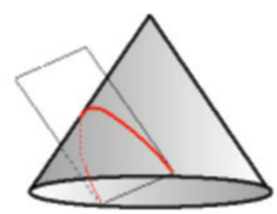
42. Kepler's second law states that as planets orbit the sun, the area of the sectors of their ellipses—two sectors are shaded in this diagram—that they trace out (measured from the sun) stays the same, as long as you measure the sectors over equal intervals of time.



What does Kepler's second law tell you about the speed of a planet with an elliptical orbit as it gets closer to the sun?

43. In a previous problem, you saw that an ellipse can be thought of as a cylindrical section. Strangely enough, an ellipse can also be thought of as a "conic section," as long as you cut the cone at a shallow enough angle.

It's also possible to slice a cone exactly parallel to its side, like this:



In this case, you don't get an ellipse as all. Rather, you get a shape called a "parabola."

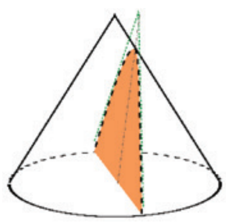
You almost certainly have experience with parabolas. It's the shape traced out by a ball when you throw it through the air. In fact, this is the approximate shape traced out by any projectile near the surface of the Earth (as long as there isn't too much air resistance). If you have a phone capable of taking slow motion video, record two of your classmates tossing a ball back and forth. Does the path of the ball resemble a parabola?

Handout 2 ▸ P.8

Problems on Conic Sections

44. If you slice a cone at anything steeper than parallel to the sides, you get yet another shape, called a "hyperbola."

What shape do you get if you slice a cone vertically downward exactly through its center? What about if you cut horizontally across the cone?



45. Can you figure out how to use a flashlight to create an image of a circle? An ellipse? A hyperbola? Why might a parabola be difficult? And what does a flashlight have to do with cones? By the way, the flashlights on most cell phones won't work well for this activity. Flashlights with a single bulb and a mirrored backing work the best.

46. As you have seen, circles, ellipses, and parabolas all are possible shapes of the paths of objects when acted upon by a gravitational force. Although Kepler was probably not aware of them, perhaps it shouldn't be surprising that "hyperbolic orbits" are possible. Given what you know about hyperbolas and their asymptotes, describe the sorts of objects that would have hyperbolic orbits.

47. For each of the following situations, choose the conic section that best describes the path of the object. In some cases, you may need to look up orbital data or the meaning of a term.

a. The moon in its orbit around the Earth
b. Comet Hale-Bopp
c. Alan Shepard's flight in the Freedom 7 capsule
d. John Glenn's flight in the Friendship 7 capsule
e. The Chelyabinsk meteor that broke up over Russia in 2013, but long before it entered Earth's atmosphere
f. Voyager 1 during its "gravitational slingshot maneuver" about Jupiter (from Jupiter's point of view)
g. A satellite in a "geosynchronous transfer orbit"
h. The New Horizons space probe, today

Teacher Resource 1

Problems on Scientific Notation — Answer Sheet

1. Try to elicit reasons and explanations from the students.

1a. One possible answer is $26 \cdot 3$. Some students will probably just answer "78." Discuss why this might obscure the process highlighted in the problem.

1b. $17b$

1c. $50c + 180d$

2. One possible example, using 1a:

$5 \cdot 3 + 6 \cdot 3 + 7 \cdot 3 + 3 \cdot 8 = 3 \cdot (5 + 6 + 7 + 8) = 26 \cdot 3$

3. 7^{300} (This example was chosen to be too large for many calculators. You may wish to challenge students with a few similarly calculator-unfriendly problems. If you use Lesson 6, it's an interesting follow-up to later show that GlowScript can handle large numbers, e.g., `print(7**40 * 7**260)`.)

7^{a+b}

4. One justification is to write out the factors

$$\frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}$$

and then show that five pairs of factors from the numerator and denominator "cancel." As for something that cannot be simplified in this way, any division problem with relatively prime bases will do, such as

$$\frac{10^{20}}{9^7}$$

5. Subtracting exponents results in

$$\frac{4^5}{4^8} = 4^{-3}$$

Try to lead students to see that this suggests the definition

$$b^{-a} = \frac{1}{b^a}$$

By the same reasoning, it's reasonable to define $b^0 = 1$. You may also wish to discuss why it's wise to impose the limitation $b > 0$.

6. $17 \cdot 8^{354} + 23 \cdot 8^{354} = 30 \cdot 8^{354}$

Any other example illustrating the process of combining like terms will do.

7. $10^6 = 1000000$

$10^{-4} = 0.0001$

Lead the students to realize that the number of digits that the decimal point shifts relative to the number 1.0 corresponds to the exponent on the 10.

8. All these numbers can be found fairly quickly with a quick Internet search.

8a. Approximately 7.4×10^9 people (in 2016).

8b. Daffodil pollen (for example) has a diameter of about 4.5×10^{-5} meters.

8c. In 2014, NASA accounted for approximately 5×10^{-3} of the federal budget (0.5%).

Teacher Resource 1

Problems on Scientific Notation — Answer Sheet

8d. The diameter of the observable universe is about 8.8×10^{26} meters (9.3×10^{10} light years). By the way, the radius of the universe (as it is usually defined) is much larger than the speed of light multiplied by the age of the universe. This is due to the fact that the universe is expanding, so that galaxies that emitted light long ago have moved much farther from us since then. See the article "How is the Universe bigger than its age?" at **https://medium.com/starts-with-a-bang/how-is-the-universe-bigger-than-its-age-7a95cd59c605#.in8jz4deq** for a good discussion.

9. It's important to do these (and perhaps additional similar examples) without a calculator first, in order to build intuition for scientific notation. E notation is less cluttered than scientific notation, and is useful for writing problems in class. Scientific notation is better when the context is more formal.

9a. 6.0E17

9b. 12.0E17 or 1.2E18

9c. 7.0E-7

9d. 0.3E6 or 3.0E5

10. Answers will vary, of course. A 13–year-old is about 4.1×10^{8} seconds old.

11. Answers will vary. A person who is 160 cm (5 feet 3 inches) tall has a height of 1.60/1.40E-12 = 1.1E12 carbon atoms.

12. The NASA page at **http://voyager.jpl.nasa.gov/where/** gives continuously updated distances for the Voyager probes. Use the page to indicate mission statistics such as the distance of Voyager 1 and Voyager 2 from the sun along with other information)

13. A passenger jet may travel at a speed of perhaps 240 meters per second. At this speed, it would take 4.5E12/2.4E2 = 1.9E10 seconds, or about 600 years to reach Neptune.

14. The volume of Earth is about $\frac{4}{3}\cdot\pi\cdot(6.3\times10^{6})^{3} = 1.0\times10^{21}$ cubic meters. A cubic light-year is about $(9.5\times10^{15})^{3} = 8.6\times10^{47}$ cubic meters. So the number of Earths that would fit (ignoring the numerous practical difficulties) is about 8.6E47/1.0E21 = 8.6E26.

Teacher Resource 2

Problems on Conic Sections — Answer Sheet

1. A search for "draw circle with paper clip" will turn up a number of videos showing the method. A video can be worth more than a thousand words.

2. Likewise, search for "draw circle with loop of string." Have students use an actual loop, rather than a single length with knots at the end, as they will later draw an ellipsis with a very similar method.

3. Some other examples of this include:

* Diameter as (a) a particular line through a circle versus (b) the length of that line.

* Weight as (a) a heavy object versus (b) the force of gravity on that object.

* AB used to refer to (a) the line segment connecting points A and B versus (b) the length of that segment, as in $AB = 5$. (Sometimes the notation $|AB| = 5$ is used for the length and $\overline{AB}$ for the segment itself.

4. It's important for students to actually do this activity, in order to get a sense of where the circumference formula comes from. The formula should not be just another thing for students to memorize. (In which case they are usually doomed to continually confuse it with the circle area formula. There's a reason they both appear on the formula sheet for the SAT math sections.)

5. This is a good mini-research problem to do for homework.

6. One viewpoint is to consider the ratio of circumference to radius, which is a constant: the same number of radiuses "fit" around a circle, no matter the size of a circle. Another viewpoint is to plot circumference versus radius. The graph is a straight line with a slope of 2π.

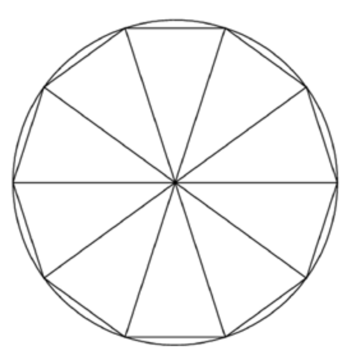
7. In one orbit, the Earth travels: $2\pi \cdot 1.5{\times}10^{11} = 9.4{\times}10^{11}$ meters.

8. The average speed of an object is its path length divided by the time of travel:

$2\pi \cdot 0.723/225 = 2.02{\times}10^{-2}$ AU/day

$2\pi \cdot 1.08{\times}10^{11}\ \mathrm{m}/(1.94{\times}10^{7}\ \mathrm{s}) = 3.50{\times}10^{4}\ \mathrm{m/s}$

9. Try to guide the students to come up with a drawing resembling the one here.



Teacher Resource 2

Problems on Conic Sections — Answer Sheet

10 and 11. These problems are intended to trap unwary students in a circular reasoning situation. You can show the first formula, given the second, or you can show the second formula, given the first. But you shouldn't be able to do both! The way out of this is to use problem 9: the sum of the areas of the triangles is

$$\frac{1}{2}b \cdot h + \frac{1}{2}b \cdot h + \cdots \frac{1}{2}b \cdot h$$

or

$$\frac{1}{2}(b + b + \cdots + b) \cdot h$$

But as the number of triangles increase, h gets closer and closer to the radius of the circle, and the sum of the bases gets closer and closer to the circumference of the circle. With a very large number of triangles, the circumference is very close to $\frac{1}{2}C \cdot r$.

12. The enclosed area is 1.64 square AU, or 3.68×10^22 square meters. The note alludes to Kepler's second law.

13. Multiplying (rather than adding) all lengths by the same amount does not change the relative size of each object, so a dilation is defined in this way. Actual measurements will depend on your printout.

14. Multiplying by a number between 0 and 1 decreases the length of an object. It's also interesting to pose the question of what multiplication by 1 does. Likewise, with a scale factor of 0. Negative scale factors can even make sense for directed line segments (vectors).

15. Scaling any linear dimension of a figure scales its area by that same factor. So the area is doubled in both cases. When both dilations are done together, of course, the area is increased by a factor of four.

16. The key is that the height of both triangles is the same, so its actual value has no effect on the ratio of their areas, which is 3/5.

17. The point of this activity (and it is strongly recommended that students actually set out physical objects representing the sun and planets) is to get a visceral feel for relative sizes. In particular, the sun is much larger than anything else in the solar system. Using a large beach ball, bean bag chair, or similar object for the sun might be better than a basketball. Or just use (part of) a large circle drawn on the classroom whiteboard or chalkboard. The video *Star Size Comparison* at https://www.youtube.com/watch?v=HEheh1BH34Q gives a good idea of how big planets and stars can get. Also interesting is the image in the article *You Could Fit All the Planets Between the Earth and the Moon* at http://www.universetoday.com/115672/you-could-fit-all-the-planets-between-the-earth-and-the-moon/.

Teacher Resource 2

Problems on Conic Sections — Answer Sheet

18. This is also a useful (and fun) activity to get a sense of the scale of the solar system.

19. Have the students calculate the scaled size of some of the planets for the activity in the previous problem. They should realize that most planets will be extremely small.

20. With Pluto, the size of the model doesn't actually increase by that much. In fact, Pluto is sometimes closer to the sun than Neptune.

21. Two successive dilations each increase the area by their respective scale factors, so doubling the base and height quadruples the area. Likewise, doubling the base and halving the height produces a triangle with the same area as the original.

22. Since doubling the radius of a circle has the same effect as scaling it in two perpendicular directions by a factor of 2, the area increases by a factor of 4. Students can also argue this using one of the area formulas for a circle.

23. Actual measurements will depend on your printout. If the original radius was b, then the scale factor is greater than 1. If the original radius was a, then the scale factor is between 0 and 1.

24. If you imagine two radiuses laid out perpendicular to each other, a dilation parallel to one of them has no effect on the other.

25. Actual measurements will depend on your printout.

26. A cut perpendicular to the base results in a rectangle. For a tilted cylinder, the shape depends on where you make the cut. It might be useful to try it with a model made of clay.

27. Since the beam is (approximately) a cylinder, an ellipse can be made by holding the pointer at an angle relative to the wall. It's useful to point out to students that the beam actually widens a bit (especially with cheaper lasers). You can lead them to realize that the actual shape of the beam is a cone.

Cheap laser pointers can be found at pet stores, where they are sold as cat toys. More powerful lasers can be ordered from science education supply companies, but they are overkill for this activity. Use extreme care with any laser pointer, as it can easily cause permanent eye damage. Seriously consider the maturity level of your students before letting them handle them in the classroom, and enforce rigid safety protocols.

Teacher Resource 2 Problems on Conic Sections — Answer Sheet

28. Actual measurements will depend on your printout. Unless the printout has been distorted in some way, students should always get the same sum. This is one way of defining an ellipse.

29. Search for "draw ellipse with loop of string" to see how to do this.

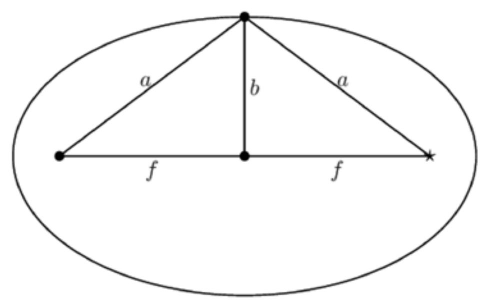
30. When the loop of string is held completely parallel to the major axis, it is easy to see that the lengths add up to the major axis itself, i.e. $2a$. The string doubles up on the outside of one focus, and that's exactly the part that's missing on the outside of the other focus.

31. The focal points merge at the center of the circle. The big radius and small radius approach each other and become just the radius of the circle.

32. Just the path of the planet is not enough to distinguish the location of the black hole. However, the planet would move faster when it is near the black hole, which would be a dead giveaway for the location of the black hole. There are several videos that show the motion of stars around the supermassive black hole at the center of the Milky Way galaxy. A search for "black hole at center of milky way" should give good results.

33. Since the aphelion and perihelion stretch across the entire ellipse, their sum is equal to $2a$ by definition.

34. The trick is to stretch the loop of string so that it forms a triangle as shown here. Since the length of the string is $2a$, the two hypotenuses are each length a. The equation follows from the Pythagorean theorem.



35. Actual measurements will depend on your printout.

36. The eccentricity of a circle is zero ($e = 0/r$). As you dilate an ellipse parallel to its major axis, the value of f approaches the value of a, even though both are growing. So the upper bound for the eccentricity is 1.

37. You can use the relation $A + P = 2a$ to calculate $a = 0.9995$. Then, since $f + P = a$, $f = 0.995 - 0.983 = 0.0165$. Finally, the eccentricity is $e = f/a = 0.0165/0.9995 = 0.0165$.

Teacher Resource 2

Problems on Conic Sections — Answer Sheet

38. The same calculation as in the previous problem gives an eccentricity of 0.207.

39. The orbit of Halley's comet is a highly elongated ellipse.

40. This mini-research project is a good opportunity to discuss what scientists actually do. In particular, be sure to point out the absolute distinction between "a plausible or scientifically acceptable general principle or body of principles offered to explain phenomena" and idle "speculation" (as defined by Merriam-Webster). These are almost opposite meanings of the same word. Unfortunately, confusion between the two has led many who should know better to apply the unfortunate phrase "just a theory" to well-established scientific theories.

41. Another good mini-research project that makes a good homework assignment. Of particular interest was the fact that in deriving his laws, Kepler made use mostly of data collected by Tycho Brahe. Brahe advocated a model in which the sun orbited the Earth, but the other known planets orbited the sun. Also interesting is that Brahe collected all his data without the use of a telescope.

42. As a planet nears the sun, it must go farther to sweep out the same area. By Kepler's second law it must do so in the same amount of time. Therefore planets must travel faster as they near the sun, and slower when they are far from the sun.

43. If you have software capable of keeping track of positions, such as Logger Pro, you can have students verify that the path is indeed a parabola. One method is to derive an equation based on three points, plot the parabola, and verify that the path is very close to the points. But the activity still gets the point across even when done only qualitatively.

44. A right cone sliced vertically through its vertex results in a triangle. (An isosceles triangle, actually.) This could be considered a degenerate hyperbola. A horizontal cut makes a circle.

45. A flashlight with a single bulb and mirror backing should illuminate an approximately cone-shaped volume when projected onto a flat surface, as long as there are no significant asymmetries in the bulb, filament, or mirror. The various conic sections can be projected on a wall by holding the light at different angles, resulting in what is essentially a "cut" through the illuminated cone of light.

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Teacher Resource 2

Problems on Conic Sections — Answer Sheet

46. An object that "falls" toward a star or planet at a very high speed will swing around the star or planet once, and continue outward, never to return again, as long as there is no collision. This type of orbit is hyperbolic. Some comets behave this way. Do a search for "hyperbolic comets" for more details.

47a. approximately circular
47b. highly elliptical
47c. approximately parabolic
47d. parabolic at launch and return, approximately circular during its orbit
47e. hypothesized to have had a roughly circular orbit in the asteroid belt, but became increasingly eccentric through gravitational perturbations. This happened to put it in the path of Earth.
47f. hyperbolic relative to Jupiter
47g. highly elliptical
47h. hyperbolic (It will leave the solar system.)

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